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ABSTRACT

This paper presents a spatial multi-product oligopoly model where there are two sets of firms. The first set considers the current firms in the market, which try to maximize their profits in a non-cooperative fashion. The second set comprehends the possible new entrants to the market. It is assumed that the new entrants have a totally competitive behavior. The equilibrium model is formulated as a variational inequality problem. The petroleum refined products market is analyzed, where existing refining companies form the first set of firms and new entrants are modeled as traders, the fastest and cheapest way (in the short term) to enter this market. A solution algorithm is proposed. Given the equilibrium prices obtained with this algorithm, a second algorithm is described in order to determine the market share of each firm from the first set, assuming they will set prices low enough in order not to attract new entrants. A numerical analysis of the Brazilian petroleum refined products market is presented.

Keywords: oligopoly equilibrium, variational inequalities, petroleum refineries.

RÉSUMÉ

Nous considérons dans cet article un modèle d'oligopole multi-produits pour deux ensembles de sociétés. Le premier ensemble contient les sociétés actives dans le marché, qui essayent de maximiser leurs profits de manière non coopérative. Le deuxième ensemble contient des sociétés qui peuvent entrer le marché. On suppose que les nouveaux entrants ont un comportement complètement compétitif. Le modèle d'équilibre est formulé comme une inégalité variationnelle. Le marché considéré est celui des produits pétroliers au Brésil. Les sociétés qui possèdent des raffineries forment le premier ensemble et les sociétés qui entrent sur le marché sont des importateurs. Un algorithme de solution est proposé. Étant donné les prix d'équilibre obtenus avec cet algorithme, un deuxième algorithme est développé pour déterminer la part du marché de chaque société du premier ensemble, en supposant que les prix sont assez bas pour décourager les nouveaux entrants. Un exemple numérique des produits pétroliers au Brésil est présenté.

Mots clés : équilibre d'oligopole, inégalité variationnelle, raffineries du pétrole.

1. Introduction

Equilibrium prices in oligopoly models can be considerably higher than the competitive equilibrium prices, depending on the number of firms active in the market. However, high prices may attract other players to this market. These players may realize positive profits with such high prices and then new equilibrium prices would be obtained after the new players join the game.

The question is how to model a spatial price oligopoly model with firms, which aim to maximize their own profits, and consider as well the possibility of new entrants, which may be attracted by the relatively high oligopoly equilibrium prices. These new entrants are expected to have a totally competitive behavior and an appropriate equilibrium model for them would be more similar to a spatial price equilibrium model than to an oligopoly equilibrium model.

In some markets, such as the petroleum-refined products, the supply can be made by producers, which use their own refining facilities, and by traders, who buy on the global market and sell locally. The producers own facilities that require capital investments, such as refineries, and are efficient in the long term. The traders can supply the market with less initial investment, but with a higher short-term cost. A trader could compete with a local refiner without requiring a large initial capital investment. However, the producer that has already invested in the refinery has smaller short-term costs than a trader, if the initial investment has been amortized. In such markets, in general, new entrants will operate as traders, while the current firms are usually producers.

The purpose of the paper is to develop a multi-product spatial price oligopoly model that considers new entrants as zero profit firms. With this model, threshold prices, which are not attractable to the new entrants, can be evaluated. Then, the demand is divided among the existing firms in an oligopoly model (without new entrants) given the threshold prices.

The paper is organized as follows. In the next section, some references about price equilibrium and oligopoly equilibrium models are presented as well as a multi-behavior traffic equilibrium model. In section 3, the multi-behavior oligopoly model is explained and formulated as a variational inequality problem. Then a solution algorithm is presented in section 4 to find the threshold prices and a second algorithm is proposed to find the market share of each current firm in the market considering these threshold prices. A numerical example based on the Brazilian petroleum refined products market is presented in section 5, in order to show the applicability of such model in a spatial multi-product market. Finally, section 6 brings some conclusions and possible extensions of this model.

2. Literature Review

One of the main assumptions of the spatial price equilibrium model (Samuelson, 1952) is that the supply of commodities occurs in totally competitive markets, where no firm can unilaterally impact the prices with a change in its production strategy. Hence, demand price equals supply price in every region and the markets clear at equilibrium.

The classical oligopoly model considers a demand market and a set F of firms supplying a homogenous product in a non-cooperative fashion. A Nash equilibrium is established when all the firms set output levels in such a way that no firm alone can increase its own profit, producing more or less products. This problem is usually formulated as a variational inequality, considering the marginal profit of each firm. More detailed definition of this model can be found in Murphy *et al* (1982), Harker (1984) and Nagurney (1993).

Harker (1986) and Nagurney (1993) present a spatial oligopoly model where the profit function of each firm considers the transportation costs from the production plants to the demand markets. Harker (1986) compares the spatial oligopoly model with the totally competitive spatial price equilibrium and with two possible monopoly situations. All the four approaches have a network structure, and can make use of network algorithms to find the equilibrium. As expected, in the examples presented, the oligopoly game profits, prices and flows are between those of the spatial price equilibrium and those of the monopoly model solution.

Okuguchi & Szidarovsky (1999) present a local (not considering spatial separation) oligopoly game with multi-product firms. Each firm has a cost function depending on the manufacturing of all products. In a general approach, a given limited input can be used to produce different products. Then, the production levels of the products can depend on each other. However, when formulating the problem as a nonlinear complementarity problem, it is assumed that the production levels do not depend on each other and, for each product, they are in the interval [0, L_p], where L_p is the firm's production capacity for product p. Then, the set of all feasible output vectors X_f, for each firm f, equals the Cartesian product of the one-dimensional intervals [0, L_p].

Equilibration algorithms for oligopoly models usually make use of diagonalization/relaxation algorithms used to solve variational inequalities. These algorithms applied to oligopoly problems have an interesting interpretation as a Cournot expectations dynamic process, (Harker, 1984 and Okuguchi & Szidarovsky, 1999). This process assumes that, in one stage, each player (firm) establish its strategy (production level) by trying to maximize its individual profit, considering that the other players will set their own strategies as the same as in the previous stage. After all the firms make their choice, the game moves to another stage and the firms play again, until no firm can improve its profit.

Harker (1988) presented a traffic network equilibrium model that considers players with different behaviors. It considers a shipper's model for the intercity freight flows problem. The shippers are the agents who purchase transportation services from carriers. The shippers are usually modeled as price takers in the transportation market and they behave according to Wardrop's user optimal principle when routing their freight over the networks of the carriers. However, Harker (1988) pointed out that not all the shippers are price takers, given that some are large freight forwarders, consolidators, and large individual shippers who routinely employ their substantial market power to negotiate better freight rates with carriers. Hence, a mixed behavior model is presented, which captures the situation where the smaller shippers (price takers) behave according to Wardrop's user optimal principle and the larger shippers each attempt to minimize their own costs in a way similar to the Wardrop's system optimal principle.

By using such an approach, a similar model can be developed to model new entrants in the oligopoly markets. These new entrants would behave according to Wardrop's user optimal principle, setting prices as if they were in a totally competitive market, to gain market share from the original players.

Novshek (1980) considered the entry of new firms in the market in a Cournot equilibrium model (spatial price equilibrium). It provides a precise statement and proof of the "Folk Theorem", which states that in competitive markets with a single homogeneous good, and free entry and exit, if firms are small relative to the market then the market outcome is approximately competitive. It is shown that if firms are small relative to the market then there is a Cournot equilibrium with free entry; furthermore, any Cournot equilibrium with free entry is approximately competitive. However, it considers that all the firms have identical technology, what is not the case of the majority of the oligopoly markets.

3. Model Formulation

Consider one product, a set K of demand markets, with separable (inverse demand functions) price functions $pr^k(q^k)$ for each $k \in K$, where q^k is the total demand in k; a set J of supply markets, with separable supply price functions $\pi_j(s_j)$ for each $j \in J$, where s_j is the quantity supplied in j; and average transportation cost functions $ct_j^k(y_j^k)$ for each pair $(j,k) \in (J,K)$, where y_j^k is the quantity shipped from j to k; the classical spatial price equilibrium problem can be written as a mathematical program:

$$\text{Maximize} \sum_{k \in K} \int_0^{q^k} pr^k(x)dx - \sum_{j \in J} \int_0^{s_j} \pi_j(x)dx - \sum_{(j,k) \in (J,K)} \int_0^{y_j^k} ct_j^k(x)dx \tag{1}$$

subject to

$$q^k = \sum_{j \in J} y_j^k, \ \forall\, k \in K \tag{2}$$

$$s_j = \sum_{k \in K} y_j^k, \ \forall\, j \in J \tag{3}$$

$$\sum_{k \in K} q^k = \sum_{j \in J} s_j \tag{4}$$

$$q^k \geq 0, \ \forall k \in K;\ s_j \geq 0, \ \forall j \in J;\ y_j^k \geq 0, \ \forall (j,k) \in (J,K). \tag{5}$$

The supply function is not stated explicitly in some models such as the PIES algorithm (Ahn, 1979), but is derived from the dual variables of the producers, through the solution of a cost minimization problem. Harker (1985) incorporates this type of cost-minimization model into the classical spatial price equilibrium problem by noting that π_j is the marginal cost function under the perfect competition assumption and that $C_j(s_j)$, the minimum total cost of producing s_j units of output, can be written as:

$$C_j(s_j) = \int_0^{s_j} \pi_j(x)dx + \text{constant}. \tag{6}$$

Hence, (1) can be rewritten as:

$$\text{Maximize} \sum_{k \in K} \int_0^{q^k} pr^k(x)dx - \sum_{j \in J} C_j(s_j) - \sum_{(j,k) \in (J,K)} \int_0^{y_j^k} ct_j^k(x)dx\,. \tag{7}$$

Instead of being totally competitive, consider now that this market is an oligopoly, where F is the set of firms in this market. q_f^k denotes the supply of firm f, $f \in F$, to demand market k, and Q^{k*}_{F-f} the optimal supply of all the other firms ($\neq f$) to the same market k. J_f is the set of supply markets where the firm f has production facilities. For simplicity, it is assumed that only firm f has production facilities in market $j \in J_f$. The profit maximization problem of firm f competing to supply this market with a Cournot-Nash behavior can be written as:

$$\text{Maximize } \sum_{k \in K} pr^k\left(q_f^k + Q^{k*}_{F-f}\right)q_f^k - \sum_{j \in J_f} C_j\left(s_j\right) - \sum_{(j,k) \in (J_f,K)} ct_j^k\left(y_j^k\right)y_j^k . \tag{8}$$

Subject to

$$q_f^k = \sum_{j \in J_f} y_j^k \ , \ \forall\, k \in K \tag{9}$$

$$s_j = \sum_k y_j^k \ , \ \forall\, j \in J_f \tag{10}$$

$$q_f^k \geq 0,\ \forall k \in K;\ s_j \geq 0,\ \forall j \in J_f,\ y_j^k \geq 0,\ \forall (j,k) \in (J_f,K). \tag{11}$$

It is assumed that firm f is a price taker in the transportation market, and then it takes ct_j^k as fixed. The firm f also takes Q^{k*}_{F-f} as fixed, given the assumption that each firm behaves in a Cournot-Nash manner.

Note that the quantity Q^{k*}_{F-f} considers the output of all the other firms, the large ones and the small ones. Then, this profit maximization problem can be used by a large firm competing against only one other large firm to supply a given product, as well as against several small and large firms. This problem can be used to represent the behavior of any large player in the market. However, to represent the behavior of the small players, the formulation (1)-(5) seems to be more appropriate. Let W be the set of all small firms in the market, which have a totally competitive behavior. Assume also that the set F contains only the large firms, i.e., F and W are disjoint, and all the firms in the market are represented by $(F+W)$. These firms in W are price-takers in the transportation market. Also, let J_W denote the set of supply markets where the firms in W have production facilities. Let $q_W{}^k$ denote the supply of the firms in W to demand market k, and Q^{k*}_F the optimal supply of the firms not in W (i.e., in F) to k. The problem of these competitive players against the large firms can be written as:

$$\text{Maximize } \sum_{k \in K} \int_0^{q_W^k} pr^k\left(x + Q_F^{k*}\right)dx - \sum_{j \in J_W} C_j\left(s_j\right) - \sum_{(j,k) \in (J_W,K)} ct_j^k\left(y_j^k\right)y_j^k \tag{12}$$

subject to

$$q_W^k = \sum_{j \in J_W} y_j^k \ , \ \forall\, k \in K \tag{13}$$

$$s_j = \sum_k y_j^k \ , \ \forall\, j \in J_W \tag{14}$$

$$q_W{}^k \geq 0,\ \forall k \in K;\ s_j \geq 0,\ \forall j \in J_W;\ y_j^k \geq 0,\ \forall (j,k) \in (J_W,K). \tag{15}$$

Consider now a multi-product problem, where the price function of the product $p \in P$ (P is the set of all products) may depend on the consumption levels of all products in all demand markets,

$pr^{pk} = pr^{pk}(\mathbf{q}_F, \mathbf{q}_W)$, where the demand vector $\mathbf{q}_F$ with components q_f^{pk}, the quantity of product p consumed in market k from firm $f \in F$, and similarly $\mathbf{q}_W$ with components q_W^{pk}, the quantity of product p consumed in market k from all firms in W. The production cost function of plant j of each firm $f \in F$ is now denoted by $C_{jf} = C_{jf}(\mathbf{s}_{jf})$, where $\mathbf{s}_{jf}$, with components s^p_{jf}, is the output vector of all products from plant j. For the plants $j \in J_W$, the production cost functions are dependent on the production of all products in the plant $\mathbf{s}_{jW}$, with components s^p_{jW}, i.e., $C_{jW} = C_{jW}(\mathbf{s}_{jW})$. $\mathbf{s}_F$ and $\mathbf{s}_W$ are the output vectors of all products from all the plants of firms in F and in W, respectively. The transportation costs depend on the entire shipment pattern, i.e., $ct_j^{pk} = ct_j^{pk}(\mathbf{y}_F, \mathbf{y}_W)$, where $\mathbf{y}_F$ has components y_{jf}^{pk}, the quantity of product p shipped to market k from plant j of firm $f \in F$, and similarly $\mathbf{y}_W$ with components y_{jW}^{pk}, the quantity of product p shipped to market k from plant j owned by one of the firms in W.

The equilibrium conditions of this mixed oligopoly equilibrium model can be written as a variational inequality problem:

Find $(\mathbf{q}_F^*, \mathbf{q}_W^*, \mathbf{s}_F^*, \mathbf{s}_W^*, \mathbf{y}_F^*, \mathbf{y}_W^*)$ such that

$$\sum_{j\in J_W}\sum_{p\in P}\frac{\partial C_{jW}(\mathbf{s}^*_{jW})}{\partial s^p_{jW}}\left(s^p_{jW}-s^{p*}_{jW}\right)+\sum_{p\in P}\sum_{j\in J_W}\sum_{k\in K}ct_j^{pk}\left(\mathbf{y}_F^*,\mathbf{y}_W^*\right)\left(y^{pk}_{jW}-y^{pk*}_{jW}\right)-\sum_{p\in P}\sum_{k\in K}pr^{pk}\left(\mathbf{q}_F^*,\mathbf{q}_W^*\right)\left(q^{pk}_W-q^{pk*}_W\right)$$

$$+\sum_{f\in F}\sum_{j\in J_f}\sum_{p\in P}\frac{\partial C_{jf}(\mathbf{s}^*_{jf})}{\partial s^p_{jf}}\left(s^p_{jf}-s^{p*}_{jf}\right)+\sum_{f\in F}\sum_{p\in P}\sum_{j\in J_f}\sum_{k\in K}ct_j^{pk}\left(\mathbf{y}_F^*,\mathbf{y}_W^*\right)\left(y^{pk}_{jf}-y^{pk*}_{jf}\right)$$

$$-\sum_{f\in F}\sum_{p\in P}\sum_{k\in K}pr^{pk}\left(\mathbf{q}_F^*,\mathbf{q}_W^*\right)\left(q^{pk}_f-q^{pk*}_f\right)$$

$$-\sum_{f\in F}\sum_{j\in J_f}\sum_{p\in P}\sum_{k\in K}\sum_{q\in P}\sum_{l\in K}\left[\frac{\partial pr^{ql}\left(\mathbf{q}_F^*,\mathbf{q}_W^*\right)}{\partial q^{pk}_f}-\frac{\partial ct_j^{ql}\left(\mathbf{y}_F^*,\mathbf{y}_W^*\right)}{\partial y^{pk}_{jf}}\right]y^{ql}_{jf}\left(y^{pk}_{jf}-y^{pk*}_{jf}\right)\geq 0\,, \qquad (16)$$

$\forall(\mathbf{q}_F, \mathbf{q}_W, \mathbf{s}_F, \mathbf{s}_W, \mathbf{y}_F, \mathbf{y}_W)$ respecting the production schedule constraints of each plant, the flow conservation constraints and non-negativity.

Using the flow conservation constraints, as (9), (10), (13) and (14), it can be shown that the production cost functions $C_{jW}(\mathbf{s}_{jW})$ and $C_{jf}(\mathbf{s}_{jf})$, $\forall j$, and the price functions $\mathbf{pr}^{pk}(\mathbf{q}_F, \mathbf{q}_W)$, $\forall p$ and $\forall k$, are all dependent only on the shipment quantities $\mathbf{y}_F$ and $\mathbf{y}_W$. Hence, the Kuhn-Tucker equilibrium conditions for (16) are:

$$\begin{cases}\dfrac{\partial C_{jW}}{\partial s^p_{jW}}+ct_j^{pk}=pr^{pk}\,,\ \text{if } y^{pk*}_{jW}>0\\ \dfrac{\partial C_{jW}}{\partial s^p_{jW}}+ct_j^{pk}\geq pr^{pk}\,,\ \text{if } y^{pk*}_{jW}=0\end{cases}, \qquad \forall\, p\in P,\ \forall\, k\in K,\ \forall\, j\in J_W\,, \qquad (17)$$

and

$$\begin{cases} \dfrac{\partial C_{jf}}{\partial s_{jf}^{p}} + ct_j^{pk} + \sum_{q\in P}\sum_{l\in K} \dfrac{\partial ct_j^{ql}}{\partial y_{jf}^{pk}} y_{jf}^{ql} \\ \qquad = pr^{pk} + \sum_{q\in P}\sum_{l\in K} \dfrac{\partial pr^{ql}}{\partial q_f^{pk}} q_f^{ql}, \text{ if } y_{jf}^{pk*} > 0 \\ \dfrac{\partial C_{jf}}{\partial s_{jf}^{p}} + ct_j^{pk} + \sum_{q\in P}\sum_{l\in K} \dfrac{\partial ct_j^{ql}}{\partial y_{jf}^{pk}} y_{jf}^{ql} \\ \qquad \geq pr^{pk} + \sum_{q\in P}\sum_{l\in K} \dfrac{\partial pr^{ql}}{\partial q_f^{pk}} q_f^{ql}, \text{ if } y_{jf}^{pk*} = 0 \end{cases} , \forall p\in P, \forall k\in K, \forall j\in J_f, \forall f\in F. \quad (18)$$

Conditions (17) mean that a plant j in J_W will supply market k with product p if the derivative of the production cost in relation to the production of p in plant j, which is the marginal production cost of p, plus the transportation costs to market k, equals the price pr^{pk} of product p in k. This equilibrium conditions for the small players are similar to the classical spatial price equilibrium conditions (Harker, 1986, and Nagurney, 1994). For the firms who have some market power, the conditions (18) mean that a plant j of firm f will supply market k with product p if the derivative of the production cost in relation to the production of p in plant j plus the marginal transportation costs to market k, $ct_j^{pk} + \sum_{q\in P}\sum_{l\in K} \dfrac{\partial ct_j^{ql}}{\partial y_{jf}^{pk}} y_{jf}^{ql}$, equals the marginal revenue, $pr^{pk} + \sum_{q\in P}\sum_{l\in K} \dfrac{\partial pr^{ql}}{\partial q_f^{pk}} q_f^{ql}$, of selling product p to market k.

4. The Solution Algorithm

One of the common approaches used to solve variational inequalities are the relaxation algorithms. These algorithms, especially for oligopoly equilibration, solve a sequence of profit maximization problems for each firm. The price functions in each firm profit maximization problem consider the supply of the other firms, assuming they will follow the same strategy of the previous iteration. The economic interpretation is that each firm assumes that the other firms will not react to its strategy, setting their strategies as the same as before. Okuguchi & Szidarovsky (1999) called this scheme as a "Cournot expectations adjustment process". Another adjustment process, called "sequential adjustment process" by Okuguchi & Szidarovsky (1999), assumes that each firm uses the last information available. Then, instead of using the strategies of the other firms obtained in a previous iteration, one can use the strategies obtained at the same iteration for the firms that had already played. Those correspond to a Jacobi scheme and a Gauss-Seidel scheme respectively.

The price functions are also relaxed in order to make the problem integrable and hence the problem can be transformed into an optimization problem. At each iteration, cross price effects are evaluated and the price functions updated. The economic interpretation of this approach is that the firms are "myopic" when setting strategies (Ahn, 1978).

A similar algorithm can be developed for this multi-behavior oligopoly model. The only difference is that for the new entrants, the profit maximization problem is substituted by a problem like (12)-(15). This algorithm is summarized below:

Mixed Behavior Oligopoly Solution Algorithm

Step 0 (*Initialization*): Obtain a feasible strategy for each firm in F, using historical data on prices, demands and supply or even solving a problem like (8)-(11) assuming the strategies of all other firms are zero production. Similarly, obtain a feasible production strategy for the firms in W, using historical data, or solving a problem like (12)-(15), assuming the strategies of all other firms are zero production. Set iteration $t = 0$.

Step 1 (*Iteration*): Solve, for each firm $f \in F$, a profit maximization problem like (8)-(11) assuming that the supply of all other firms is given by

$$Q^{pk}_{(F+W)-f} = \sum_{g \in F, g=1}^{f-1} q^{pk}_g(t) + \sum_{g \in F, g=f+1}^{|F|} q^{pk}_g(t-1) + q^{pk}_W(t-1). \tag{19}$$

Solve, for the set W of small firms, a problem like (12)-(15) assuming that the supply of the large firms in F is given by

$$Q^{pk}_F = \sum_{g \in F} q^{pk}_g(t). \tag{20}$$

After solving (8)-(11) for each firm in F, and (12)-(15) for W, update the price function for each p and k with the new prices for product $q(\neq p)$ in sub region k, capturing cross price effects.

Step 2 (*Convergence test*): If $(\mathbf{q}_F(t) - \mathbf{q}_F(t\text{-}1)) < \epsilon$, some arbitrary small tolerance, for all firms in F, and $(\mathbf{q}_W(t) - \mathbf{q}_W(t\text{-}1)) < \epsilon$, some arbitrary small tolerance, for W, then stop. Otherwise, set $t = t + 1$, and go to step 1.

The prices obtained with this algorithm are the "threshold" prices to make it interesting for new entrants to join the game. Prices above this allow a new entrant to make a positive profit. On the other hand, any new entrant would make a negative profit with lower prices.

The prices and supplies, which are the solution of this equilibrium model, are obtained under the assumption that all possible new entrants are present in the game. The firms that are currently supplying the market demand would use such a model to determine prices low enough so that new entrants will not find it profitable to enter the market. However, by setting lower prices, the demand for each current firm will actually be larger than that predicted by the model, since the new entrants would not be present. So, while it appears that setting lower prices is not optimal in the short term, it may be a very good long-term strategy, since it protects the market share of the current firms.

Given the threshold prices obtained with the above algorithm, what would be the supplies of the current firms to the demand markets? The firms will try to maximize their own profits considering the threshold prices and the strategies of the other firms. If they supply too much, the prices will fall and so will the profits. On the other hand, if they supply too little, the prices will rise and may become higher than the threshold prices, attracting new firms to the market, and once again the profit will fall. Hence, the total supply to each demand market should be equal to the "threshold demand", which correspond to the threshold prices. But how will this demand be shared among the current firms?

If the threshold prices are fixed then a linear programming problem of the total joint profit of the industry can be formulated. The solution of this problem is probably degenerate, since some markets may be equally profitable for two different firms. Hence, the solution of this problem may not be unique. This formulation cannot capture the competitive behavior of the firms in an oligopoly market. However, its solution gives the optimal supply pattern of a monopolistic firm, or even a coalition of all the current firms in the market. For the case of non-cooperative oligopoly markets, such a linear programming formulation would not give a realistic solution, due also to the extreme point property of the solution.

In order to capture the non-cooperative nature of the firms in an oligopoly, an oligopoly equilibration algorithm with adapted price functions is proposed, based on diagonalization algorithms used to find oligopoly equilibrium (Harker, 1984, Pompermayer *et al*, 2002). This algorithm tries to find oligopoly equilibrium prices and supplies in such a way that the equilibrium prices equal the threshold prices found in the first model. This is achieved by changing the slopes of the price functions, for each demand market and product, pivoting them at the points defined by the threshold prices and their respective threshold demands. This method gives much more realistic supplies than the obtained with the linear problem of the total joint profit of the industry, as will be seen in the next section.

5. A Numerical Example

The algorithms presented in the previous section were applied to the Brazilian market of petroleum-refined products. This application is based on the data available for the month of April 2001. At that time, the Brazilian market was not open to competition, and the Government established the ex-refinery prices for the main products. Price elasticities were estimated based on the time series data from 1977 to 1997. Linear price functions are fitted to data, which include cross price effects.

Petroleum refined products are usually produced in refineries. The profit maximization problem of refining companies is not simple. It must consider the special characteristics of the refining process. The production of each product is closely related to the production of the others. The quantity that will be produced of each product depends on the crude oil characteristics, the complexity of the refinery (i.e., the ability to transform heavy hydrocarbon chains into lighter ones), and on the operating mode chosen. Petroleum refining companies usually make use of linear programming models to optimize the operation of the refineries and prepare the production schedules. Then, the profit maximization problem of refining companies must consider this cost minimization problem formulated as a linear programming problem. More detailed description about this profit maximization problem is given in Pompermayer *et al* (2002).

Three aggregated products are considered: light distillates, medium distillates and heavy distillates[1]. Table 1 shows the price and cross price elasticities for these three products, estimated for the Brazilian market. There are fifteen internal demand sub-regions where the demand functions for these products were calibrated. These sub-regions have been defined to correspond

[1] The main product in light distillates is gasoline, diesel oil in the medium distillates, and fuel oil in the heavy distillates.

to storage terminals and distribution centers used by distribution companies, where the competition in supplying refined products is expected to occur.

Table 1: Price elasticities used in the simulation.

Price elasticities	Gasoline	Diesel	Fuel oil
Gasoline	-0.15	0.03	0.00
Diesel	0.05	-0.12	0.04
Fuel oil	0.00	0.03	-0.40

Hence, a linear price function of the form

$$pr^{pk}\left(d^{pk}\right) = \frac{pr_0^{pk}}{\varepsilon^{pp}} \cdot \frac{d^{pk}}{d_0^{pk}} + \frac{pr_0^{pk}\left(\varepsilon^{pp} - 1 - \Delta pr^{q \neq p,k}\right)}{\varepsilon^{pp}}, \tag{21}$$

is obtained for each product p and sub-region k, and is dependent only on the total demand d^{pk} for the product p in the sub-region k. ε^{pp} is the price elasticity of product p. pr_0^{pk} and d_0^{pk} are known price and demand for product p in sub-region k, in a given period of time. For this application, these prices and demands are based on the data available for the month of April 2001, and are listed in the tables 2 and 3 below, for light distillates only. The term $\Delta pr^{q \neq p,k}$ captures cross price effects between different products, using the prices obtained in the previous iteration in the algorithm and the cross price elasticities ε^{pq}. It is defined as:

$$\Delta pr^{q \neq p,k} = \sum_{q \in P, q \neq p} \varepsilon^{pq} \frac{\left(pr^{qk} - pr_0^{qk}\right)}{pr_0^{qk}}. \tag{22}$$

Three refining companies can supply any of the internal demand sub-regions, as well as export to an external sub-region. Transportation costs to any of the sub-regions are evaluated by considering the mode used, and the capacity of each mode and terminal, for example, the size of vessels. Two of these companies operate one refinery each, and buy crude oil on the global market. These are labeled Refiner 2 and Refiner 3 in the Tables below. The other company, the larger one (labeled Refiner 1), operates 10 refineries and several crude oil production facilities. It supplies more than 90% of the existing refining capacity in Brazil. Figure 1 shows a map of Brazil with the refineries, terminals and the demand sub-regions considered in this example.

Figure 1: Brazil, demand sub-regions, refineries and terminals.

Seven crude oil types are considered: three produced in Brazil, and four from other countries that usually trade crude oil with Brazil. Acquisition costs for the "foreign" crude oils are estimated based on prices published on the market and on the transportation costs to each refinery. Crude oil production costs were estimated by considering the type of exploitation (on-shore / off-shore).

Each refinery may operate in two modes: "maximum gasoline" and "maximum diesel". Therefore, conversion rates from each crude oil into each refined product were estimated for each refinery by using a given operating mode. This estimation considered the complexity of each refinery and the characteristics of the crude oil, especially density. It was used in estimating refining costs as well.

It is assumed that traders, the new entrants, may also supply refined products to any of the 15 internal demand regions, acquiring products basically in two foreign points, Argentina and Venezuela. The traders purchase the products at a fixed price. This assumption is based on the fact that volume acquired by each trader must be small compared to the global market, not influencing the prices in the market. However, if the supplying capacity of the points where the traders can purchase products is not considerably big, supply price functions, increasing with the quantity purchased, must be evaluated. It is also assumed that transportation costs are constants. Specially when representing the new entrants in the petroleum refined market as traders, the assumption of fixed purchase prices is adequate since it will give the price at which the first new entrants can supply the demand markets.

Table 2 below presents the supplies for light distillates obtained with the Mixed Behavior Oligopoly Solution Algorithm and compare them with a pure oligopoly scenario (not considering new entrants) where two traders try to maximize their own profits (not a totally competitive behavior) competing with the current refiners already in the market. Table 2 has in the rows the

sub-regions in Brazil and the external one "Export". The second column presents the actual demand with regulated prices for each sub-region in April/2001. The following columns present the supply by each firm in the MBO and pure oligopoly situation. The profit of each firm is also given. Table 3 presents the equilibrium prices of the multi-behavior model and compares them with the regulated prices practiced in April/2002 and with the equilibrium prices of the pure oligopoly scenario with two traders. These equilibrium prices of the multi-behavior model are practically equal the sum of the fixed purchase prices in the foreign points plus the transportation costs to each demand market.

Table 2: Light distillates consumption and supplies [m^3].

	d_s^u	MBO - Multi-Behavior Oligopoly					Pure Oligopoly					
Sub-regions	(Apr/2001)	Ref. 1	Ref. 2	Ref. 3	Traders	Total	Ref. 1	Ref. 2	Ref. 3	Trader 1	Trader 2	Total
PA/AP	28,077	610	0	225	27,385	28,220	7,813	335	493	7,201	7,201	23,044
AM/RR/RO/AC	36,631	1,198	70	375	34,825	36,468	10,555	186	399	9,357	9,357	29,853
MA/PI	26,422	509	0	145	25,990	26,644	7,339	271	420	6,830	6,830	21,689
CE/RN	53,861	1,248	61	500	52,134	53,943	15,080	549	856	13,832	13,832	44,147
PE/PB/AL	75,001	1,690	23	662	73,268	75,643	20,929	897	1,353	19,238	19,238	61,655
BA/SE	82,436	3,210	597	1,288	76,881	81,976	24,071	482	965	20,861	20,861	67,239
RJ/ES	183,231	7,438	2,523	4,898	167,870	182,729	52,952	1,820	3,733	45,805	45,805	150,117
MG	188,306	8,985	2,613	5,073	170,022	186,694	55,537	1,288	3,270	46,855	46,855	153,805
SP W	144,365	6,880	2,223	3,215	130,886	143,203	42,818	1,292	1,918	35,939	35,939	117,907
SP central	200,000	9,139	3,235	4,413	181,976	198,762	58,872	2,346	3,022	49,731	49,730	163,701
SP NE	300,000	12,517	4,577	6,620	275,315	299,029	87,467	3,596	4,885	74,947	74,948	245,843
RS	154,286	4,203	710	1,007	150,095	156,015	44,237	1,471	1,378	40,035	40,035	127,156
PR/SC	207,805	5,751	0	1,427	202,780	209,958	59,884	866	1,897	54,134	54,134	170,915
MS/MT	47,442	1,908	616	891	43,709	47,124	13,165	1,650	1,823	11,257	11,257	39,151
GO/DF/TO	118,993	5,379	1,738	2,513	108,461	118,091	34,543	2,078	2,567	29,165	29,165	97,517
Sub-total	1,846,856	70,662	18,986	33,253	1,721,597	1,844,499	535,260	19,128	28,980	465,185	465,186	1,513,739
Export	-	2,062,625	0	0	-	2,062,625	1,586,018	0	0	-	-	1,586,018
Profit [10^6 $]	-	1,170.600	7.514	8.815	≈ 0	1,186.929	2,651.807	41.491	46.632	1,111.765	1,111.767	4,963.462

It can be seen that the demand in the multi-behavior model is mainly supplied by the traders, who have a totally competitive behavior. The Refiner 1 exports almost all of your production, in order to maximize its own profit. This show the possible future scenario that the current refiners could face if they maintain the strategies of setting prices based only in the current firms in the market. The market might change from a situation as the "Pure Oligopoly", where only two traders are in the market, with considerably high prices (see Table 3), to a situation with several new traders attracted by the high prices, making the market almost totally competitive, cutting down the profits of the current refiners.

Table 3: Light distillates prices [$/m^3].

Sub-regions	pr_0^{tr} (Apr/2001)	MBO	Pure Oligopoly
PA/AP	507.70	481.63	1,349.98
AM/RR/RO/AC	492.20	493.04	1,331.22
MA/PI	505.50	472.05	1,343.13
CE/RN	501.20	482.04	1,340.09
PE/PB/AL	507.10	473.32	1,340.49
BA/SE	488.00	492.32	1,315.58
RJ/ES	489.20	487.67	1,302.96
MG	485.30	498.66	1,303.70
SP W	486.20	498.67	1,305.58
SP central	490.60	497.66	1,310.93
SP NE	490.60	489.66	1,306.76
RS	487.80	439.72	1,283.56
PR/SC	485.60	440.73	1,284.06
MS/MT	576.20	588.68	1,500.13
GO/DF/TO	512.60	525.02	1,362.59

Using the threshold prices obtained with the Multi-behavior Oligopoly Solution Algorithm, the second algorithm mentioned at the end of the previous section was applied to the same scenario of "Pure Oligopoly", with two traders, i.e., three refiners, two traders and no new entrant. Given the price functions are linear functions, the changing on slopes was made increasing the absolute values of the price elasticities in Table 1. The algorithm considered prices slightly higher than the threshold prices in order to give some profitability to the two traders, so they will not leave the market. Table 4 presents the supplies of each firm for light distillates, their profits, and the slightly higher prices than the threshold prices. The supplies of the linear problem of the total joint profit of the industry (LP model), considering the threshold prices, are also presented. The traders supplied nothing of light distillates in the solution of the LP model, they only supplied medium distillates, and so they are not shown.

Table 4: Light distillates consumption and supplies [m^3], with fixed prices.

	d_a^u	Pure oligopoly with fixed prices (changing prices slopes)							LP model		
Sub-regions	(Apr/2001)	Ref. 1	Ref. 2	Ref. 3	Trader 1	Trader 2	Total	Prices[$]	Ref. 1	Ref. 2	Ref. 3
PA/AP	28,077	23,778	0	0	1,494	1,494	26,766	486.87	28341	0	0
AM/RR/RO/AC	36,631	33,796	0	0	1,001	1,001	35,798	496.31	36763	0	0
MA/PI	26,422	21,059	0	0	1,895	1,895	24,849	478.00	26696	0	0
CE/RN	53,861	43,955	0	0	3,490	3,490	50,935	487.98	54329	0	0
PE/PB/AL	75,001	62,606	0	0	4,433	4,433	71,472	478.67	75782	0	0
BA/SE	82,436	78,288	0	0	1,472	1,472	81,232	494.75	82643	0	0
RJ/ES	183,231	150,213	0	19,544	4,747	4,747	179,251	491.24	135827	18986	28980
MG	188,306	159,080	0	17,581	3,920	3,920	184,501	501.86	188254	0	0
SP W	144,365	134,919	2,254	0	2,431	2,431	142,035	501.36	144350	0	0
SP central	200,000	183,030	7,022	0	3,460	3,460	196,973	500.39	200286	0	0
SP NE	300,000	274,915	6,276	0	6,684	6,684	294,559	492.87	300974	0	0
RS	154,286	135,292	0	0	7,295	7,295	149,881	444.25	156964	0	0
PR/SC	207,805	168,325	0	0	14,400	14,400	197,125	446.75	211122	0	0
MS/MT	47,442	42,255	1,216	187	1,271	1,271	46,200	593.21	47415	0	0
GO/DF/TO	118,993	109,776	2,218	0	2,361	2,361	116,715	528.24	118963	0	0
Sub-total	1,846,856	1,621,286	18,986	37,312	60,354	60,354	1,798,292	-	1808708	18986	28980
Export	-	337,603	0	0	-	-	337,603	414.80	30342	0	0
Profit [10^6 $]	-	1,767.971	8.008	9.992	1.499	1.499	1,788.969	-	1,830.336		

As can be seen, the Refiner 1 supplies much more relatively to the two traders than in the "Pure Oligopoly" scenario presented above, since the profitability of the two traders decreased considerably. Another interesting result is the higher profit obtained by the Refiner 1 in this scenario than in the MBO scenario, since the number of competitors is considerably smaller. This shows that setting the threshold prices is a good strategy, protecting the market share and profit in the future. Even for the two small refiners, the profit increased, but no so much as for the Refiner 1, since they had already reached production capacity in the MBO scenario. Especially for the Refiner 3, the total amount of light distillates is different from the MBO scenario since it changed the production schedule for the three products, producing more light than medium and heavy distillates in this scenario.

The solution of the LP model is highly degenerate. The degeneracy was especially marked for the markets 'RJ/ES' and 'RS'. Coincidently, the refineries of the two small refiners (Refiner 2 and Refiner 3) are located in these two markets. The total joint profit in the LP model is greater than the profit in the oligopoly model with fixed prices. This is to be expected since the LP model corresponds to a monopolistic firm.

The quadratic optimization problems, solved at each iteration in the algorithms presented in the previous section, were solved by ILOG CPLEX in a SUN Ultra Sparc Station, 300 MHz CPU, 128 MB RAM. The MBO Solution Algorithm converged to a solution in less than one minute. The second algorithm, which solves a sequence of oligopoly models adapting the slopes of the price functions iteratively, solved the problem in approximately 30 minutes.

6. Conclusions

An oligopoly model that considers the possibility of new entrants into the market was presented. An application to the Brazilian petroleum refined products market has shown that firms should not set strategies considering only their short run profit maximization problems, since it could attract other firms to the market. These new firms could jeopardize the long-term market share and possibly profits of the current firms.

This multi-behavior spatial multi-product oligopoly model was formulated as a variational inequality problem. Two solution algorithms were presented, one to determine the threshold prices, which make the market profitable for new entrants, and another to determine the equilibrium oligopoly supplies of the current firms given the threshold prices.

The analysis of the profit function of each firm in the oligopoly market may lead to a leader-follower oligopoly model, where one of the firms has the power to set the prices while the others simply react to these prices by changing their production schedules. In this situation, the leader firm tries to maximize their own profit and set prices by considering the reaction of the other firms present in the market and the possibility of new entrants. This is the subject of a forthcoming paper.

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